Analysis of Sampled Optical Phase-Lock Loops

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***Abstract*—Sampled optical phase-lock loops are analyzed using the Z-transform. It is found that the finite sampling rate will result in an effective delay limiting the available stable gain of the loop. The analysis is applied to three example applications, optical wavelength synthesis from a pulsed reference, linear phase-tracking receivers and coherent optical receivers.**

I. INTRODUCTION

Optical Phase-Lock Loops (OPLLs) are used in a range of applications including carrier recovery in coherent optical receivers [1], wavelength synthesis from an optical frequency comb reference [2], offset locking for RF or THz frequency synthesis [3], synchronization of laser arrays [4] or linear phase demodulation in analog optical links [5]. For many of these applications, standard PLL control theories using the Laplace-transform to evaluate closed-loop behavior provide an accurate description of loop stability and performance [6].

In a sampled OPLL the phase information is obtained in discrete measurements. It is difficult to generate a rigorous analysis including the effect of sampling rate using standard Laplace-transform PLL theory. Instead the Z-transform, commonly used for analysis of sampled data systems [7] generates a much fuller model of these systems. In this paper the Z-transform is applied to sampled OPLLs and the relation between laser linewidth, loop delay and sampling rate are derived. Applications where this analysis apply include comb line selection by locking to a pulsed reference, phase locking in sampling downconversion optical systems and coherent phase-locked receivers where phase marker bits are used for LO laser synchronization.

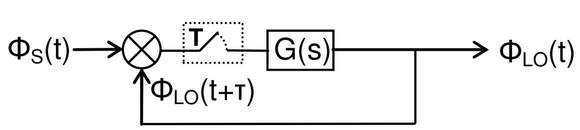

Figure 1. Basic operational schematic of the optical phase-lock loop.

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II. THEORY

The basic operation of an optical phase-lock loop is illustrated by the schematic in Fig. 1. This illustrates a loop in which the LO laser phase forms the output signal. The open-loop transmission, *G(s)*, is given by the phase detection gain, the loop filter function and the LO laser tuning efficiency. A first order loop gain function contains one integration, typically within the frequency tuning of the LO laser. The loop transmission can be reduced to:

$$G(s) = is_0 \cdot e^{-s\tau} \big/ s \tag{1}$$

where s_0 represents to the unity-gain frequency. The $e^{-s\tau}$ term accounts for the feedback group delay, τ. A second order loop contains a second integration and the loop transfer function can be expressed by:

$$G(s) = \frac{-s_n^2\left(1-2i\xi\, s/s_n\right)\cdot e^{-s\tau}}{s^2} \tag{2}$$

s_n corresponds here to the natural frequency and ξ is the loop damping factor. In standard OPLL theory, no sampling occurs, as represented by a closed gate in Fig. 1. The output phase, φ_{LO} is then related to the input phase, φ_S as:

$$\varphi_{LO} = \frac{G(s)}{1+G(s)}\varphi_S \tag{3}$$

Loop stability is ensured by keeping open loop gain below unity at the frequency where the phase crosses -180° due to the delay term.

We can use the modified Z-transform of eq. 1 and eq. 2 to evaluate a sampled loop with delay [7]. The first order loop transmission is then expressed as:

$$G(Z) = \frac{-iS_0T}{Z^n(Z-1)} \tag{4}$$

and the second order loop:

$$G(Z) = \frac{-S_n^2T^2}{Z^n(Z-1)^2}\left[1+(Z-1)\left(\frac{\tau_d}{T}+\frac{2i\xi}{S_nT}\right)\right] \tag{5}$$

Z is the Z-transform variable given by $Z(f) = exp(2\pi ifT)$ where T is the sampling period. It can be observed that the effect of delay in the loop is expressed differently than for a standard OPLL. The delay is here expressed in the form $\tau = nT + \tau_d$ where the integer number of periods in the loop have the largest impact on loop behavior. Again, the output phase, φ_{LO} is then related to the input phase, φ_S as:

$$\varphi_{LO} = \frac{G(Z)}{1+G(Z)}\varphi_S \tag{6}$$

The validity of the sampled model is confirmed by increasing the sampling rate and a convergence between the sampled and baseband OPLL performance is observed (Fig. 2). At higher sampling rates, a close correlation to the continuous loop response is seen at lower frequencies. As the frequency increases, the sampled loop gain finds a minimum at $f=1/2T$ before it starts to increase to reach a maximum at $f=1/T$. This can be understood in terms of Nyquist sampling where the relation $G(f) = G(f+1/T)$ should be valid.

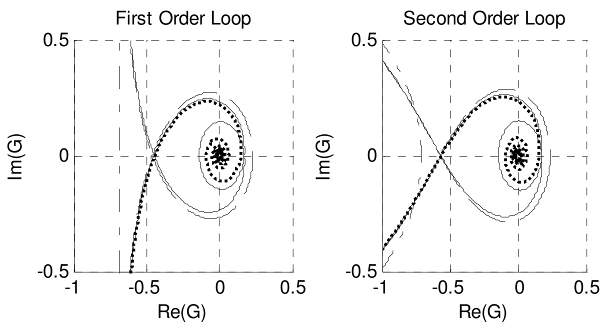

Figure 2. Nyquist diagram for first and second order loop gain function for non-sampled operation (dotted line), pulse period; $T = 2\tau$ (dash-dot line), $T = 2\tau/3$ (dashed line) and $T = 2\tau/5$ (solid line).

III. APPLICATION EXAMPLES

The above analysis can be applied to a number of applications for a sampled OPLL for analysis not readily available using standard Laplace theory. Examples include phase-locking to a pulsed reference, sampling downconversion systems and coherent receivers utilizing phase marker bits. In the following, a second order loop with critical loop damping ($\xi = 1/\sqrt{2}$) and 10 dB loop gain margin for stability will be assumed.

A. *Phase Locking to a Pulsed Reference*

Carrier-envelope locked femtosecond frequency combs provide a highly stable optical frequency reference [2]. By phase-locking a laser to any of the comb lines, a stable optical frequency synthesis system is formed. The accuracy to which this can be achieved is dependent on the pulse repetition rate. The phase information obtained from comparing the local oscillator (LO) laser phase to that of a first pulse is used to tune the LO phase to match the phase of a second pulse. This generates an effective delay in the loop corresponding to the pulse period, with the attributed feedback gain restrictions to maintain stability.

The total phase error variance is calculated by:

$$\sigma^2 = \int \frac{d\nu}{2\pi f^2}\left|\frac{1}{1+G(Z(f))}\right|^2 df \tag{7}$$

where $d\nu$ is the linewidth of the beat term between reference and LO laser, here equal to that of the LO laser assuming that the phase jitter of the pulsed laser is negligible. For clarity, a Lorentzian linewidth has been assumed here. More elaborate linewidth models can also be used in the above formula.

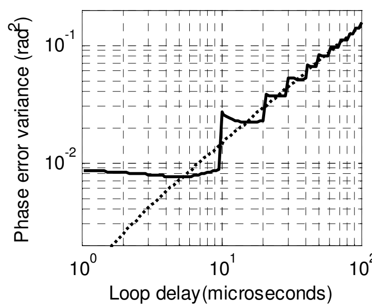

Figure 3. Integrated phase error variance versus loop delay for standard (dotted line) and sampled (solid line) OPLL. 1kHz laser linewidth and 1 MHz sampling rate is assumed.

Figure 3 shows the integrated phase error variance for a 1-kHz linewidth laser versus delay in the feedback loop, both for a standard OPLL and with 1 MHz sampling rate. At long delays the sampled loop performance approaches that of the standard loop. At very low delays, the sampled loop performance is determined by the effective delay introduced by the sampling rate, where further decrease in physical loop delay does not translate into improved performance. Figure 4 illustrates this point by showing the integrated phase noise as a function of the ratio of pulse frequency and LO laser linewidth. If a 1-MHz repetition rate is assumed, it is seen that a 1-kHz linewidth laser will result in a 0.01 rad^2 integrated phase error variance, even a high-quality 1-Hz linewidth will generate a modest 10^{-5} rad^2 variance. This illustrates a difficulty in transferring a highly stable pulsed optical reference to a highly stable CW optical frequency.

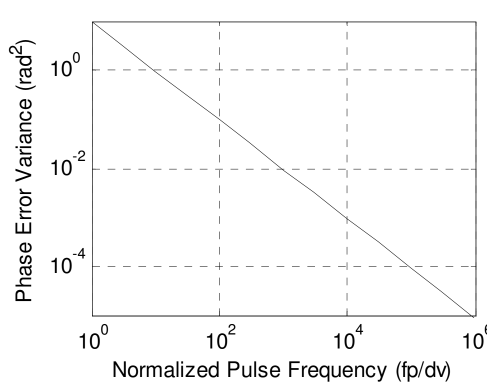

Figure 4. Integrated phase error variance versus laser linewidth normalized by pulse repetition rate for phase locking to a pulsed reference.

B. Downsampling OPLL receiver

Optical phase-tracking loops can be used to generate linear optical phase demodulation, as outlined in Fig. 5 [5]. The tracking phase is related to input phase by the loop gain in eq. 3. If a linear reference modulator is used to track the phase of an incoming optical signal, the reference drive voltage is also linearly related to the input phase if the gain is high. The delay in the tracking loop will limit the stable loop gain as the signal frequency increases, as shown by the baseband trace in Fig. 4. A compact tracking loop with 20 ps latency is assumed, similar to [8].

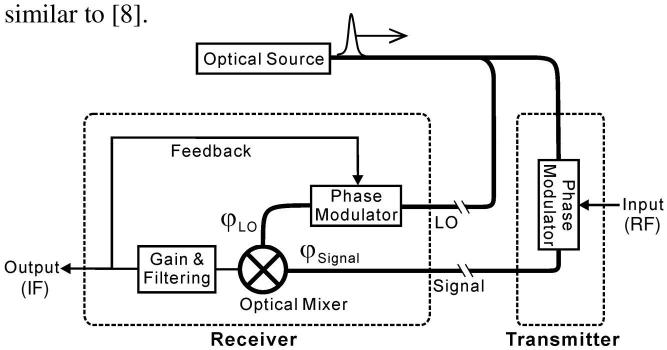

Figure 5. Concept schematic of linear coherent receiver with feedback [5]. Thick lines: optical link; thin lines: electrical link.

One method to extend the frequency range of this receiver is to use optical downconversion of a received RF signal to IF within the bandwidth of the tracking loop. It has been shown that to preserve linearity in the tracking loop, a pulsed optical signal must be used [9]. The reference modulator now tracks the downsampled received signal with a phase given by eq. 6. The available loop gain as a function of input signal frequency is shown in Fig. 6 assuming downsampling to 500 MHz. It is found that this type of receiver has the highest performance in baseband operation below 1 GHz or sampled above 10 GHz. The flattening in the frequency dependence of the sampled gain at higher frequencies represents the transition between latency limited by pulse rate and limited by physical delay in the loop.

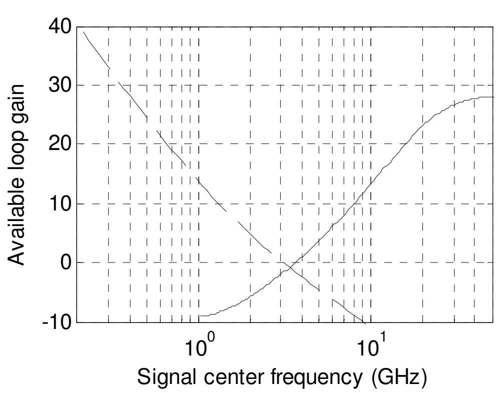

Figure 6. Available feedback gain in a linear tracking optical phase lock loop demodulator. The dashed line represents available stable feedback gain versus frequency of baseband tracking using a CW optical carrier. The solid line represents available stable feedback gain using a pulsed optical carrier and downconverting to 500 MHz.

C. Coherent Receiver

This analysis is well suited for the analysis of certain types of optical phase-lock loop receivers. The phase error variance of the LO laser can be expressed by:

$$\sigma^2 = \int \frac{d\nu}{2\pi f^2} \left| \frac{1}{1+G(f)} \right|^2 + \frac{q(P_S + P_{LO})}{2RP_{LO}P_S} \left| \frac{G(f)}{1+G(f)} \right|^2 df \quad (8)$$

The second term in the integration is the phase noise resulting from conversion of receiver shot-noise which can be significant at low power. It has been shown that the loop bandwidth is in fact a tradeoff between the need for phase tracking and the requirement to filter shot-noise [10]. Figure 7, thin solid line shows an example where this is observed. Even though here the loop delay is assumed to be negligible, the optimum natural frequency, f_n of the loop is limited to ~100 MHz assuming a 100 kHz beat linewidth and 100 photons per bit at 10 Gbps. In a sampled loop, where the phase sample from one bit is integrated and used to track the phase of the following bit, as shown by the thick solid trace in Fig. 7, the loop becomes unstable at f_n > 2.2 GHz due to the added effective delay.

For more complex modulation formats such as n-QAM, the carrier phase is not easily recovered. One option is to use designated phase marker bits for LO laser synchronization, in effect representing a sampled feedback loop with reduced sampling rate. The dot-dash, dashed and dotted thick traces in Fig. 7 shows a sampling rate of every 3rd, 10th and 30th bit, respectively. Two effects are observed from the reduced sampling rate: the loop oscillation is pushed to lower frequencies and the signal-to-noise ratio is reduced, as less received power is used to synchronize the phase. The penalty is higher than predicted from the equivalent reduction in SNR using standard non-sampled theory (thin lines) as a result of instabilities introduced in the loop.

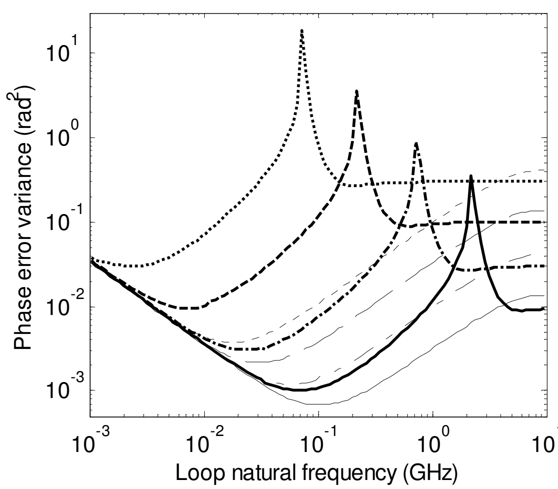

Figure 7. Integrated phase error variance versus laser linewidth normalized by pulse repetition rate for phase locking to a pulsed reference (thick lines). The solid line represents sampling every bit, dot-dash every 3rd, dashed every 10th and dotted every 30th. Comparative curves using standard Laplace theory are also included (thin lines).

Taking these plots as an illustrative example, we see that we require every 3rd bit to carry the phase information to reach a phase-noise limited BER of 10^{-9} for 16-QAM modulation ($\sigma^2 \approx 2.7e\text{-}3\ rad^2$). To require every 10th bit to carry the phase information while retaining the same phase error, we will need to reduce the linewidth from 100 kHz to 10 kHz.

IV. Conclusion

With the application of the Z-transform to optical phase-lock loop a range of applications with a sampled discrete feedback signal can be analyzed. It is found that the sampled feedback signal is attributed with an effective delay term that limits the available stable gain in such system. Examples include phase locking to a pulsed reference where the finite pulse rate fundamentally limits the ability to correlate the locked phase to that of the pulsed reference. A second example is in linear optical phase demodulation where a feedback receiver structure is used. Using a pulsed optical carrier, the received RF is downconverted to fall in the loop bandwidth. It is found that the best performance is obtained at a high RF to IF ratio. The last example involves a phase-locked coherent receiver where the phase of received signal and LO laser are synchronized using phase marker bits at regular intervals. It is found that instabilities introduced in the loop from reduced sampling rate degrade performance more than a simple SNR analysis would suggest.

Acknowledgment

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